# Applecross Senior High School

### Semester One Examination, 2019

### Question/Answer booklet

# MATHEMATICS

**SOLUTIONS**

**SPECIALIST**

**UNIT 1**

## Section Two:

## Calculator-assumed

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| Student number: In figures |  |  |  |  |  |  |  |  |  |  |

 In words

 Your name

## Time allowed for this section

Reading time before commencing work: ten minutes

Working time: one hundred minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,
correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Workingtime (minutes) | Marks available | Percentage of examination |
| Section One:Calculator-free | 8 | 8 | 50 | 52 | 35 |
| Section Two:Calculator-assumed | 13 | 13 | 100 | 98 | 65 |
|  |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.
Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section Two: Calculator-assumed 65% (98 Marks)

This section has**thirteen (****13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 (5 marks)

In the diagram below, is the midpoint of .



If and , express the following in terms of and .

(a) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct expression |

(b) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  indicates correct method✓ correct expression |

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  indicates ✓ correct expression |

(c) . (2 marks)

Question 10 (8 marks)

Points and have coordinates and respectively. Determine

(a) . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct vector |

(b) . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct vector✓ magnitude |

(c) , where is a unit vector in the direction . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ indicates unit vector✓ expression for result✓ correct vector |

(d) The coordinates of point , given that . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expression for result✓ correct coordinates |

Question 11 (7 marks)

(a) In the diagram below (not drawn to scale) and lie on a circle and and are tangents to the circle. If and , determine the size of .

 (3 marks)

 

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  ✓   |

(b) Quadrilateral is such that , and .

(i) Sketch a diagram to show this information. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct diagram |

(ii) Show that is cyclic and hence determine the size of . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ use isosceles triangle for  uses supplementary angles for cyclic correct size of  |

Question 12 (8 marks)

(a) Show that the vectors and are perpendicular. (2 marks)

|  |
| --- |
| **Solution** |
| Hence perpendicular as scalar (dot) product is . |
| **Specific behaviours** |
| ✓ uses dot product explains result |

(b) Determine, to the nearest degree, the angle between the vectors and .

 (2 marks)

|  |
| --- |
| **Solution** |
| Using CAS: Or:  |
| **Specific behaviours** |
| ✓ indicates method correct angle |

(c) The vectors and are perpendicular, where is a constant. Determine the value(s) of and the corresponding pair(s) of vectors. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses dot product to form equation solves equation states one pair of vectors states both pairs of vectors |

Question 13 (8 marks)

(a) and lie on a circle with diameter (diagram not to scale). Determine the size of when . (2 marks)

 

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ uses angle in semi-circle for  correct value |

(b) and lie on a circle (diagram not to scale). Secant cuts the circle at and is a tangent to the circle at . Given that and , determine the size of and the size of . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓    |

 

(c) and lie on a circle of radius mm (diagram not to scale) and mm. Determine the size of angle , to the nearest degree. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ completes diagram uses trig ratio for half-angle correct angle |

 

Question 14 (9 marks)

The parts of this question refer to the word AERIFICATION. It has different consonants and vowels, some of which are repeated.

(a) Determine the number of ways that different consonants chosen from the letters of the word can be arranged in a row. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct number |

(b) Determine the number of ways that all the letters of the word can be arranged in a row.

 (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ attempts to account for repeated letters correct number |

(c) Determine the number of ways that all the letters of the word can be arranged in a row if the vowels must all be adjacent. (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ counts vowels as single group counts ways to arrange vowels correct number |

(d) Determine how many letter permutations (e.g. TFI, IRI, etc) can be made using the letters of the word. (3 marks)

|  |
| --- |
| **Solution** |
| All different: Two A's and one other: Two I's and one other: Three I's: Total:  |
| **Specific behaviours** |
| ✓ attempts to consider separate cases correct number containing A's and I's correct total |

Question 15 (8 marks)

(a) In the diagram below (not drawn to scale) and lie on the circle with centre . Determine the size of angles , and given that and . (4 marks)

 

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct  equation for ✓ correct ✓ correct  |

(b) Write the converse of the theorem that states the opposite angles of a cyclic quadrilateral are supplementary. (1 mark)

|  |
| --- |
| **Solution** |
| When opposite angles in a quadrilateral are supplementary, the quadrilateral is cyclic. |
| **Specific behaviours** |
| ✓ correct statement |

(c) Prove by contradiction that the converse you wrote in (b) is true. Start by assuming that there is a quadrilateral that *does* have supplementary opposite angles but is *not* cyclic, such as shown below. (3 marks)



|  |
| --- |
| **Solution** |
| From assumption, .But from regular theorem, .Hence , but this is impossible (as and would then be parallel and triangle would not exist). Thus, our original assumption must be wrong, and the converse must be true. |
| **Specific behaviours** |
| ✓ uses assumption develops contradiction explains contradiction and makes deduction |

Question 16 (7 marks)

Three forces and act on a point in a plane.

The forces are, N and .

(a) Determine the magnitude of the resultant force and the direction, to the nearest degree, that the resultant makes with the vector . (3 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ resultant correct magnitude correct angle |

When , the forces are in equilibrium.

(b) Determine the values of the scalar constants and for equilibrium to occur. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
|  equation using -coefficients equation using -coefficients solves for  solves for  |

Question 17 (8 marks)

(a) A set of cards is numbered from to . Determine the minimum number of cards that must be selected to ensure that at least cards in the selection have the same last digit. Justify your answer using the pigeonhole principle. (3 marks)

|  |
| --- |
| **Solution** |
| Let pigeonholes be digits and pigeons be the last digit of number on card.Then fill all pigeonholes with pigeons, a total of pigeons.The next pigeon will fill one of the pigeonholes with pigeons, and so the minimum number is . |
| **Specific behaviours** |
| ✓ defines pigeons and pigeonholes clear explanation correct number |

(b) Eight different books sit on a shelf, one of which has a hardcover and the rest softcovers. A student is told they can take away as many of them as they like but must not leave empty handed. Determine how many different selections can be made

(i) of exactly books. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct number |

(ii) altogether. (2 marks)

|  |
| --- |
| **Solution** |
| Choose either up to all books: |
| **Specific behaviours** |
| ✓ uses property of Pascals triangle correct number |

(iii) that include the hardcover. (2 marks)

|  |
| --- |
| **Solution** |
| Choose hardcover and then up to others: |
| **Specific behaviours** |
| ✓ indicates method correct number |

Question 18 (8 marks)

Relative to the origin, and have position vectors and respectively.

Particle is initially at and moves with a constant velocity of ms-1.

(a) Calculate

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct speed |

(i) the speed of . (1 mark)

(ii) the position vector of after seconds. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct position |

(iii) the distance of from after seconds. (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ vector  correct distance |

(b) Determine how long after leaving that is m from . (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ expression for  expression for  equation using distance correct time |

Question 19 (7 marks)

 is a trapezium with parallel and in the same direction to .

(a) Sketch a labelled diagram of . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct diagram |

(b) Show that . (2 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ splits and  groups vectors that make  |

(c) lies on and lies on so that . Use a vector method to prove that is a trapezium. (4 marks)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ vectors for and  obtains without and  obtains in terms of  obtains in terms of  |

Question 20 (7 marks)

Farm lies km away from farm on a bearing of . A helicopter leaves farm at am to fly to farm . The helicopter can maintain a speed of kmh-1 and there is a steady wind of kmh-1 blowing from the north.

Determine the bearing that the helicopter should steer and the time of its arrival at farm , to the nearest minute.

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ diagram showing vectors and resultant✓ equation using sin rule for  value of ✓ correct bearing✓ equation using sin rule for  value of ✓ correct arrival time |

Question 21 (8 marks)

Determine how many of the integers between and inclusive are

(a) divisible by . (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct number |

(b) divisible by or . (3 marks)

|  |
| --- |
| **Solution** |
| LCM:  |
| **Specific behaviours** |
| ✓ number divisible by  indicates use of inclusion-exclusion✓ correct number |

(c) divisible by or but not both. (1 mark)

|  |
| --- |
| **Solution** |
|  |
| **Specific behaviours** |
| ✓ correct number |

(d) divisible by or but not . (3 marks)

|  |
| --- |
| **Solution** |
| LCM's:  |
| **Specific behaviours** |
|  divisible by  divisible by ✓ correct number |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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